Does chaos reign?

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In the last few years, the scientific community has found itself in the grip of a new theory which on some accounts has completed the scientific revolution of the twentieth century. Relativity, quantum theory and now chaos have combined to change the face of classical science[1]. Yet one striking aspect of the development of chaos has been the way in which it has broken through the specialist compartmentalism of science being the by-product of research encountering common problems in a number of fields with applications claimed to range from the fibrillation of the heart to the behaviour of the stock exchange. This would seem to suggest that it has revealed a methodological shortcoming at the heart of science not merely a breakthrough in one science or another.

Its relevance within these pages is two-fold. First is the question of the misuse to which it has been put and the mysticism it has generated. In this context, it has directly spilled over into social, economic and political life and as a consequence has found its way into the media and political discussions. Here, it has often been presented as the cause of things inexplicable or unforeseen, or as concurring with irrational aspects of ideological viewpoints.

But also the theory has considerable significance for statistical modelling since it raises questions concerning the approach to modelling itself. The 'traditional' approach is to attempt to identify, piecemeal, the object of study in a way that this or that piece of methodology is adequate for this or that process. This may be described as the static approach. Chaos is part of a dynamic approach (but not the only possibility) which focuses on how quantitative change gives rise to qualitative change.

It is the contention of this article that chaos needs to be demystified. That it does not serve ideological ends - has little relevance to whether a market system is the natural order of things. That it has posed important questions concerning the limitations of static methodologies and has exposed a particular methodological failing at the heart of science, although it has not fully addressed these questions. That it is not a panacea despite its frequent presentation as such and in particular it is not synonymous with the dynamic systems approach.

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1. For a selection of discussions see: Gleick, J., (1987), *Chaos*, Sphere Books; Stewart, Ian, (1990), *Does God Play Dice?*, Penguin Books; Thompson, J. M. T. and Stewart, H. B., (1986), *Nonlinear Dynamics and Chaos*, Wiley; Bartlett, M. S., (1990), *Chance or chaos*, J. R. Statist. Soc. **A**, p3; Kamminga, H., (1990), *What is this thing called chaos?*, New Left Review, p181; McGarr, P., (1990), *Order out of chaos*, International Socialism, 48.

Historical panaceas?

It is important to locate the philosophy underlying contemporary science historically. The nineteenth century, as historian E. H. Carr points out, was the 'age of facts' in which science believed in a clockwork world seeking after exact cast-iron laws, albeit with inconsistencies[2]. Yet by the beginning of the twentieth century despite monumental successes cracks began to appear in a number of areas. A particular problem which emerged in many fields has been how to cope with the situation in which quantitative change gives rise to qualitative change. This is the context in which the 'qualitative systems approach' which includes chaos was developed, because it is patently where static quantitative analysis is of little use.

The area where there have always been difficulties is in dealing with 'exceptions'. They may be merely a wild observation, or may they may signal a qualitative change of some sort. Formal analysis is happy with the former, but the latter is an anathema since it has no way of dealing with them. Exceptions may be exceptional but have the habit of occurring time and again and when they do occur it is of crucial importance to be able to deal with them and recognise them for what they are. This is a more general problem than chaos has presented, although its central thrust may be seen as giving 'exceptions' a systemic basis and as such requiring a fundamental re-thinking of the limitations of quantitative data-assessment.

During the 1970s, in a wide variety of fields, it was found that there were situations in which quantitative change gave rise to qualitative change in the form of sharp discontinuities. This 'discovery' was hailed in the media as the greatest innovation since Newton's *Principia*. It was a new panacea which accounted for qualitative change - known as catastrophe theory - as the world was surrounded by catastrophes in the same way that it is now surrounded by chaos. But within a short time controversy raged with some suggesting that it had nothing to offer. However, it was a considerable scientific advance which posed new questions and had a number of important applications, one of particular note here is the ability to model the pressures on decision-makers to expediency[3]. Indeed, whether or not one finds catastrophe theory to their taste, the problems it attempted to confront are still with us. Nevertheless, it never attempted to be a panacea, but only give some justice to aspects of a system which could not be provided by quantitative modelling which in turn showed the problems confronting modelling perceived as merely a quantitative data-assessment exercise.

2. An interesting critique of this approach can be found in Henri Poincar's *La Science et l'hypothèse*, (1903). (English translation Dover 1954). Particularly since Poincar provided much of the theory and insights associated now with chaos.
3. Harrison, P. J. and Smith, J. Q., (1980). *Discontinuity, decision and conflict* (with discussion). Bayesian Statistics, Valencia University Press.

The contrast with chaos is raised here first because both are grounded in a qualitative dynamic systems approach, but chaos shows other more complicated behaviour is possible as well, and both have been given the status of panaceas within a decade of each other. But more importantly both have focused attention away from the static conception of continuity in nature and social life.

What chaos is not

First, it is not a form of mysticism associated with unpredictable, inexplicable occurrences without rhyme or reason. Second, it is not valid to slip from chaos in its strict scientific sense as a form of randomness given certain conditions within the context of an evolving system to chaos as is commonly used in the English language as a form of uncontrollable, unpredictable disorder. It has already been used in this context, for example, by political commentators to promote the virtues of the market[4]. Finally, it is not valid to take chaos outside the context in which it is to be used as though it simply provides a model for irregularity when it occurs by applying it statically at that point[5].

What is chaos?

The place to start understanding chaos is within the context of a qualitative dynamic system. This may be contrasted with quantitative dynamic modelling which models a time-dependent, yet continuous process. The former shows that there are situations in which quantitative change can give rise to qualitative change. Thus, as is well-known, a quantitative rise in temperature beyond boiling point will result in a qualitative change. But chaos shows that this can occur in a variety of situations which were previously assumed to result merely in quantitative change. The theory of chaos charts how certain systems behave as changes occur. This shows that the system may behave as anticipated and follow the estimated quantitative model, but then as quantitative changes occur affecting model parameters (while retaining the qualitative model form), behaviour may become (unexpectedly) periodic or even completely random (chaotic).

It has been pointed out that in some cases this has occurred owing to 'simplifying' assumptions which with hindsight have proved to be invalid. Even if this were true, it tends to hide rather than elucidate the central difficulty of the quantitative static approaches. For if the aim is to model an unchanging static situation, the problem merely becomes one of misspecification which is to miss the point. Moreover, the

4. W. Rees-Mogg, *The Independent*, 14th October 1989. Denis Healey interview in *Marxism Today*, July 1990.
5. For example Bartlett, op.cit.

proponents of the theory emphasise that this randomness is not rooted in the uncertainties of information flows or controlling factors, but in the process itself. This too is well-known in some modelling contexts where randomness is 'in nature' rather than simply in the system.

Demystifying chaos

Many of the problems which have characterised chaos are fairly well-known to statistical modellers. This is so because they are products of a form of randomness. These have certainly occurred in the new context of deterministic dynamic systems, and while this might be emphasised, the phenomenon is not new. This discussion is necessary to present the chaos as accessible to a rational scientific method and to firmly reject some of the more popular overtones associated with it.

1. Randomness is not 'chaotic' (as used in the English language), it can be completely modelled mathematically; it has a rational and scientific basis. A well-known introduction to (mathematical) statistics by D. V. Lindley[6] defines statistics as 'the study of random phenomena' so the aim is to deductively describe various structures and their characteristics, or to inductively infer phenomena are of a particular type. Deterministic functions are replaced by probability distributions.

2. Deterministic chaos. The core of the theory shows that randomness can be generated within deterministic systems. Yet this is far from unknown as many calculators have random number generators, based on deterministic routines. Simulation is also a well-known technique for 'relative' chaos in which data is generated to follow a specified distribution such as the normal (and it might be noted that 'absolute' chaos also follows a specified distribution).

3. Chaos shows that systems can be highly sensitive to initial conditions, and since precise initial conditions are hard to specify exactly this process is unavoidable. This means that long-term forecasting comes up against formidable problems. However, the suggestion that forecasters, weather or otherwise, cannot forecast with any accuracy on a day a year or so ahead was pre-chaos a truism and none in their right mind would attempt to do so! Forecast uncertainty rises rapidly with time even if things are expected to be well-behaved.

4. It also shows that many systems have a tendency to settle down to a particular state (attractor) which may be random (strange attractor). This too is familiar ground for the statistical modeller although s/he would not regard it as 'strange'.

6. Lindley, D. V., (1965), *Probability and Statistics*, vol. 1, p1, Cambridge University Press.

It is possible to appreciate that the shift from determinacy to randomness might well have been seen as chaotic to scientists used to more law-like outcomes. Rutherford, for instance, in more certain days once described science as physics or stamp collecting - precisely quantifiable or nothing. Nevertheless, had chaos been discovered by statisticians it is more likely to have been given a more sober title - perhaps, the statistics of deterministic systems?

The limits of chaos

The limits to chaos need to be placed within a theoretical and methodological context. The former assessing the assumptions underlying its occurrence and the latter concerning its location in the general qualitative modelling approach.

In theoretical terms, chaos is a form of randomness which is generated under certain conditions in deterministic non-linear[7] models within the context of an evolving system[8]. The first point is that it will only occur under specific conditions irrespective of whether it has an appropriate model form, and moreover, not all non-linear systems exhibit chaotic behaviour. Second, a deterministic function is, for example, a formula and a non-linear function is one which has powers in it[9]. Any process adequately described linearly - and this is a broad class - cannot exhibit chaos. Third, while chaos can occur in the one-dimensional discrete cases, it requires at least three dimensions in the continuous case. Fourth, so-called 'non-linear' modelling is not as intractable as has been presented (leading necessarily to linear approximations).

In methodological terms, chaos has pursued one possibility in the context of a dynamic qualitative systems approach. It has looked at how qualitative change may occur within the context of a given system. The more general framework might look at how changes occur as the system itself changes in terms of its variables, cyclic factors, variance components and qualitative form.

7. The meaning of 'non-linear' here is not its usual meaning in statistical modelling. In the latter, it is only models in which parameters are not linear that would described as such (eg. $A+Bx+Cx^2$ is linear, but Ax^B is not).
8. This is based on a definition given by a meeting of the Royal Society, although there has been a change of emphasis here (Stewart, op. cit. p17).
9. The 'standard' chaos generator is the logistic function $x_{t+1}=kx_t(1-x_t)$ which is such a non-deterministic 'formula'.

Beyond chaos

Traditional dynamic quantitative modelling has addressed situations in which time-dependent fruition effects are of significance, but these still assume that the objective of modelling is process continuity within a subperiod and to ascertain the constant underlying relationships (the optimal parameter values)[10]. The aim being to ascertain the optimum quantitative model so as the situation changes in the process of development the aim is to find the next optimum quantitative model and so on. What is missing is the connection between the two - the process of development particularly if the development is not smooth. It is here that the static quantitative approach is revealed as most unsatisfactory. Moreover, while chaos has the merit of showing that kinds of qualitative change are possible within a specified system, it does not fully address the question. The system itself may change qualitatively in terms of its variables, its variance components, its qualitative form. But this takes us some way beyond chaos to the dynamic qualitative approach as a methodology which is not limited by any particular model form.

This also reveals a deeper problem - how to deal with development. To look at the data, carry out an analysis, obtain a plot and so on and then on this basis alone construct a model clearly will not do. There have been attempts to overcome this difficulty by 'structural' modelling the quantitative change and as such is limited. It does not ask, but why does it change? It is now necessary to deal coherently with the system not merely its quantitative outcomes. It is necessary to ask what are the behavioural objectives and constraints within the system?[11]

Conclusion

In rectrospect the question that chaos has brought home is why in so many fields has science been found wanting? The answer that chaos gives us is that science has attempted to statically model and placed itself in a straitjacket and as such it has revealed its blindness to situations in which there is systemic development. Indeed, while chaos is no panacea it has focused attention on broader methodological questions. Chaos has the merit of challenging the 'clockwork' vision of science and has certainly ensured a re-thinking of fundamental questions. In part it has addressed questions which in the past were overlooked or avoided and it provides considerable insights into the inadequate role of quantitative modelling in research particularly its emphasis on regularity and its inability to look at how systems develop.

10. One of the most extreme versions of this is the work in econometrics of David Hendry and others who attempt to stringently test model assumptions and parameter stability to find the optimum model.
11. For a critique and alternative see Harrison, J. and West, M., (1989), *Bayesian Forecasting and Dynamic Models*. Springer-Verlag.

The focus on development means that one should not see one set of techniques for the quantitative and one for the qualitative: a system encompasses both. Continuity and change are not alternatives, but aspects of a process in development. Continuity may give rise to change which may express itself in a new form of continuity which may give rise to further change and so on. This changes the perception of reality and therefore how it is to be understood, described and modelled. This illustrates why the development of modelling methodologies which are able to capture how a system functions and how it is likely to develop is an essential task.